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SUMMER RESEARCH TECHNICAL REPORT

Development of a Lumped Element Circuit Model for Approximation of Dielectric Barrier Discharges

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14. ABSTRACT
This work presents a circuit model for calculating the total energy dissipated into neutral species for pulsed direct current (DC) dielectric barrier discharge (DBD) plasmas. Based on experimental observations, it is assumed that nanosecond pulsed DBDs, which have been proposed for aerodynamic flow control, can be approximated by the two independent regions of a homogeneous electric field. An equivalent circuit model is developed for the homogeneous region near the exposed electrode, i.e., the ?hot spot,? based on a combination of a resistor, capacitors, and a zener diode. Instead of fitting the resistance to an experimental data set, a formula is established for approximating the resistance by modeling a plasma as a conductor with DC voltage applied to it. Various assumptions are then applied to the governing Boltzmann kinetic energy equation to approximate electrical conductivity values for weakly ionized plasmas. The model is compared with experimental data sets of the total power dissipated by a plasma to validate its accuracy.

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Abstract

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Student Bio

I am currently an undergraduate student attending the University of Florida (UF), majoring in both nuclear engineering and physics. At UF, I am a member of the Applied Physics Research Group headed by Prof. Subrata Roy, where I conduct research on plasma physics. In May of 2011, I completed his second year of studies at UF and was lucky enough to have an opportunity to work with plasma actuators on a project of interest for the U.S. Army. Upon completion of my undergraduate education, I hope to pursue the subject of applied plasma physics in graduate school in order to obtain a doctoral degree.

1. Introduction/Background

The need for improved control over aerodynamic flow separation has increased interest in the potential use of plasma actuators. The inherent advantages of plasma actuator flow control devices include minimal size and weight, lack of moving parts, cost efficiency, and low drag penalties. However, the actuators that affect the flow via directed momentum transfer are not effective at Mach numbers associated with most subsonic aircraft applications. Recently, Roupassov et al. (*1*) demonstrated that pulsed plasma actuators, in which energy imparted to the flow appears to effectively control flow separation, seem to be suitable at Mach numbers (M ≈ 0.3) beyond the capabilities of the current plasma-induced momentum-based approaches.

Given the fundamental differences between the novel pulsed discharge approach and the more conventional momentum-based approaches, there is a need to develop an effective and efficient model for the energy delivered to the flow by the plasma. Once calculated, that value can be input to a computational fluid dynamics solver as an energy source term resulting in a coupled fluid/plasma dynamics model. Multiphysics models of this type are required in order to study detailed flow characteristics. However, detailed numerical calculations that are primarily used for isolated plasma simulations are not suitable for a variety of coupled fluid/plasma dynamic studies because of their excessive computational expense. To address this issue, efficient circuit element models have been introduced to approximate the complex dynamic processes within plasmas. However, models such as those by Orlov et al. (*2*) rely on empirical constants that are tuned to experiments which are not applicable to nanosecond pulsed discharges. To date, an approximate model of nanosecond pulsed plasma actuators has not been developed. This paper deals primarily with establishing a flexible model that could be implemented as an approximation for the energy dissipated within a plasma for any pulsed direct current (DC) dielectric barrier discharge (DBD) configuration. Among the other fundamental goals in this paper is to establish a deeper understanding of the background processes that occur within plasma and incorporate that knowledge into the model.

2. Theoretical Development

One of the primary assumptions in creating this model is that a small volume near the exposed electrode can be idealized as consisting of a homogeneous energy density. The "hot spot," observed by Roupassov et al. (*1*), plays an important role in the power dissipated into the air and thus the resulting fluid dynamics. In this study, only the spatially homogeneous hot-spot region was considered. Therefore, the model presented in this paper consists of a single network containing a resistor, capacitors, and a diode.

As shown in figure 1, circuit elements that were used to model the plasma include an air capacitor C_a, a dielectric capacitor C_d, a resistor R_f, and a zener diode D_f. The air capacitor represents the capacitance between the dielectric surface and the exposed electrodes. The dielectric capacitor represents the capacitance between the dielectric surface and insulated electrodes and is proportional to the thickness of the dielectric layer. Thus the dielectric layer in the form of both its thickness and the value of its dielectric constant plays an important role in determining the effectiveness of the plasma actuator. Finally, the zener diode, introduced by Orlov et al. (*2*), is used in the model to enforce an energy threshold value below which plasma will not form.

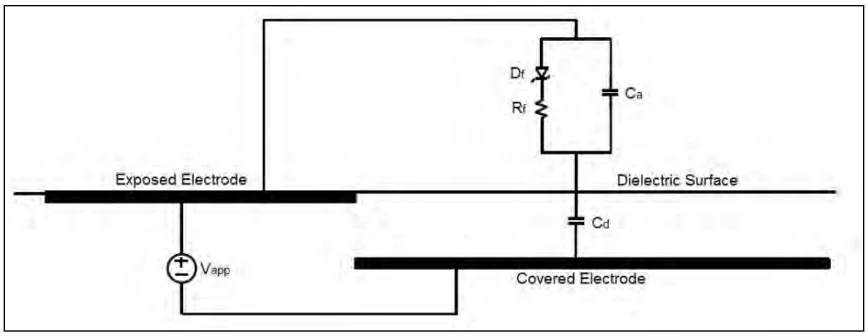

Figure 1. Electric circuit model of a dielectric aerodynamic plasma actuator.

Since a uniform charge distribution along the top of the dielectric is assumed, the typical asymmetric plasma actuator geometry featured in figure 1 can be simplified to a homogeneous symmetric region. This assumption results in a one-dimensional (1-D) model, and any variation along the horizontal direction depicted in figure 2 is neglected.

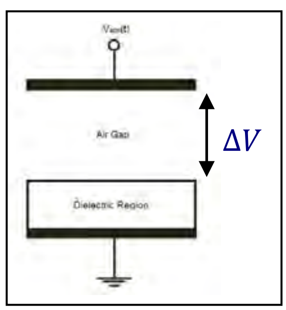

Figure 2. Region of homogeneous potential, i.e., "hot spot."

As displayed in figure 1, the lumped element circuit is a function of the two capacitance values, C_a and C_d. In this model, the air is treated as both a conductor to generate a physical relationship for the resistance R_f and a parallel plate capacitor to generate C_a. An advantage of modeling the plasma as a conductor in addition to a parallel plate capacitor is that it generates a physical relationship for the resistance, R_f—a value that is traditionally empirically determined. The air gap capacitor can be modeled as (*3*)

$$C_a = \frac{\epsilon_0 \epsilon_a A_a}{h_a}, \tag{1}$$

where A_a is the cross-sectional area of the air and h_a is the approximate height of the plasma region of interest. As displayed in figure 3, A_a is the product of the spanwise length of the actuator z_a, and l_a is the chordwise distance from the exposed electrode to the end of the dielectric region.

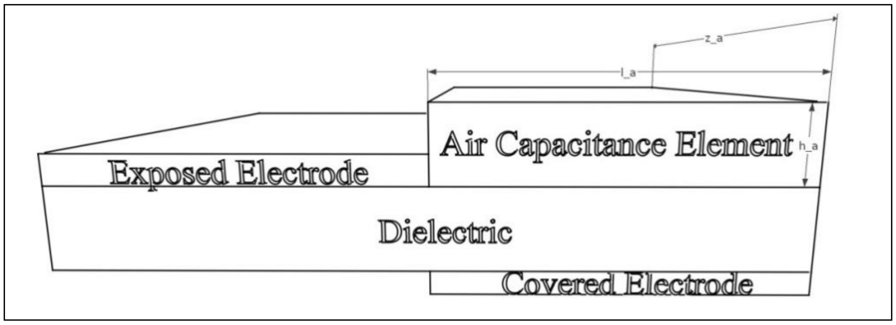

Figure 3. Sketch of the air capacitive element.

The capacitive element corresponding to the dielectric can be modeled as (*3*)

$$C_d = \frac{\epsilon_0 \epsilon_d A_d}{h_d}, \tag{2}$$

where A_d is the cross-sectional area of the dielectric capacitive element and h_d is the height of the dielectric barrier layer. As displayed in figure 4, A_d is the product of the spanwise length of the actuator z_a and d_d is the width of the dielectric region. Treating the plasma as a conductor, the resistance for DC voltage is proportional to the electrical conductivity, σ_p, as well as A_a, and h_a and can be given as (*4*)

$$R_f = \frac{h_a}{\sigma_p A_a}. \tag{3}$$

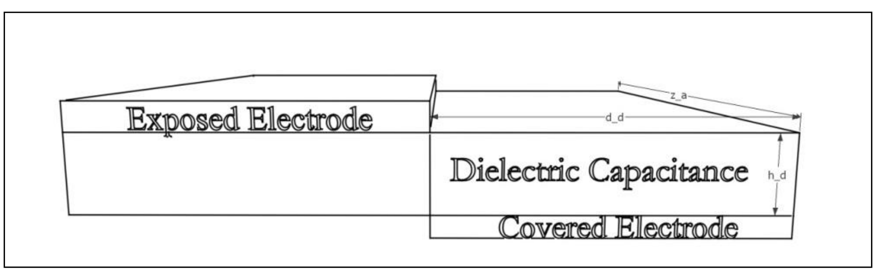

Figure 4. Sketch of the dielectric capacitive element.

Starting from Kirchhoff's laws (*3*), the governing differential equation for the voltage drop experienced by the air gap ΔV is given by

$$\frac{d\Delta V(t)}{dt} = -\frac{dV_{app}(t)}{dt}\left(\frac{C_a}{C_a + C_d} - 1\right) - \kappa \frac{\Delta V(t)}{R_f(t)(C_a + C_d)}, \tag{4}$$

and

$$\kappa = \begin{cases} 1 \; if \; |\vec{E}| > \overrightarrow{|E|}_{crit} \\ 0 \; if \; |\vec{E}| < \overrightarrow{|E|}_{crit} \end{cases}, \tag{5}$$

where V_{app} is the applied voltage and κ is the contribution from the zener diode. If the electric field magnitude, given as

$$|\vec{E}| = \frac{|\Delta V|}{h_a}, \tag{6}$$

is greater than some threshold value, then κ takes on a value of one, otherwise it is zero to signify that plasma has not formed. For nanosecond high-voltage pulses, the applied electric field will virtually always be over the critical breakdown field (*1*), thus κ can be set equal to one for the case considered in this study.

The energy imparted to neutral species from the plasma can be calculated from knowledge of the voltage drop over the air gap by

$$E(t) = \int_0^t \frac{\Delta V}{R_f(t)} dt. \tag{7}$$

To effectively calculate the resistance governed by equation 3, an expression must first be developed for the electrical conductivity of the plasma. This value is one that traditionally requires a numerical approach. To simplify the problem to a point where an analytic formulation can be used, numerous simplifying assumptions were used and are described in the following paragraphs.

For any plasma, the resulting electric current is composed of two primary terms: the current from electrons and that from ions. As the drift velocity, $\boldsymbol{w_e}$, which represents the velocity induced by an electric field, is significantly higher in a nonequilibrium plasma for electrons compared to ions, the current density can be approximated as only the portion from electrons as long as the number densities, N_e and N_i, are approximately the same (*4*). When a form of the generalized Ohm's law is used, the current density vector $\boldsymbol{J}$ and plasma conductivity, respectively, can be written as

$$\boldsymbol{J} \approx -eN_e\boldsymbol{w_e} = \sigma_p \boldsymbol{E}, \tag{8}$$

and

$$\sigma_p = e(N_e\mu_e + N_i\mu_i), \tag{9}$$

where μ_i and μ_e represent the ion and electron mobilities, respectively. Much like equation 8, the electrical conductivity relation can be simplified using the concept of quasineutrality, which is defined as having approximate equal number densities of both ions and electrons. Thus, as μ_e is typically approximately three orders of magnitude larger than μ_i for any plasma that is not in thermal equilibrium, if N_e is at least the same order of magnitude as N_i, it is a good assumption to approximate the electrical conductivity as only coming from electrons (*4*). Quasineutrality itself is a typical assumption that is valid as long as the plasma being modeled is far away from the cathode to avoid the boundary layer in plasma physics called the sheath.

Since a pulsed DC voltage is assumed, the activation of the external electric field will follow the voltage waveform as a step function. Thus two expressions will be required for the σ_p, where the first is valid for the period when a constant external electric field is applied over the peak of the voltage pulse, as shown in figure 5, from 13–40 ns, and the second when the external voltage is zero. For the portions of the voltage waveform that the voltage is zero, the power is also zero according to Ohm's law, and thus the conductivity during this time is of no importance.

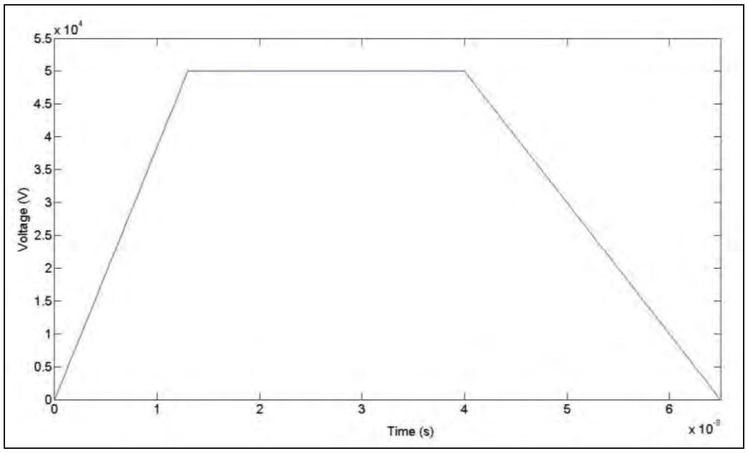

Figure 5. Input voltage, V_{app} vs. time.

Given an assumption of a constant electric field during the peak, an analytic formulation can be generated for the electrical conductivity with knowledge of $\boldsymbol{w_e}$ (*5*) by starting with the Boltzmann equation for electrons written as

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \frac{\boldsymbol{F}}{m} \cdot \frac{\partial f}{\partial \boldsymbol{r}} = \frac{df}{dt}, \tag{10}$$

where df/dt is typically called the collision integral and refers to the variation in the number of particles within some phase space control volume. In order to approximate this, a relaxation time, τ, is introduced, which is defined as the time taken for the system to be reduced to an equilibrium distribution function. This approximation can be given as $\tau \approx (N_a \sigma_{ea} v)^{-1}$, where σ_{ea} is the collision cross section, v is the average collision velocity, and N_a is the number density of atoms (*5*). According to Smirnov (*5*), an analytic formulation is obtained by assuming that the force term can be given as the Lorentz force,

$$\boldsymbol{F} = -e\boldsymbol{E}\exp(-i\omega t) - \frac{e}{c}(v \times \boldsymbol{B}), \tag{11}$$

where ω is the frequency the electric field, $\boldsymbol{B}$ is the magnetic field and assumed to be directed along the z-axis (vertical), and $\boldsymbol{E}$ is assumed to belong only in the x(horizontal)-z plane. Taking the tau approximation of electron kinetic energy equation and integrating over the electron velocity, we can then write the equation of motion for an electron as

$$m_e \frac{d\boldsymbol{w_e}}{dt} + m_e \frac{\boldsymbol{w_e}}{\tau} = -e\exp(-i\omega t) - \frac{e}{c}(v \times \boldsymbol{B}). \tag{12}$$

Substituting the cyclotron frequency $\omega_H = eB/m_e c$, $a_x = -eE_x/m_e$ and $a_z = -eE_z/m_e$, the following scalar equations representing the different components of equation 12 can be produced:

$$\frac{dw_x}{dt} + \frac{w_x}{\tau} = a_x \exp(-i\omega t) - \omega_H w_y, \tag{13}$$

$$\frac{dw_y}{dt} + \frac{w_y}{\tau} = \omega_H \omega_x, \tag{14}$$

and

$$\frac{dw_z}{dt} + \frac{w_z}{\tau} = a_z \exp(-i\omega t). \tag{15}$$

Equations 13–15 can be solved to obtain three component expressions for the electron drift velocity. If a constant electric field and a negligible magnetic field are assumed, ω and ω_H can be set equal to zero and the resulting vector equation can be obtained:

$$\boldsymbol{w_e} = -\frac{e\boldsymbol{E}\tau}{m_e}. \tag{16}$$

When equation 16 is solved in conjunction with equation 8 for σ_p, the following expression independent of the applied electric field can be obtained:

$$\sigma_p = \frac{N_e(t)e^2}{m_e N_a \sigma_{ea} v}. \tag{17}$$

In equation 17, σ_{ea} is a function of the electron temperature, T_e, and can be obtained for various molecules found in air (*6*). As shown in equation 17, the inputs required for this approximation include N_e , the number density of electrons, N_a, and T_e. Among these values, N_a is assumed to be constant in time as the number density of atoms is significantly higher than that of free electrons. T_e is solved for quantity from the energy transport equation and changes primarily with reduced electric field strength, E/N_a. Results for the electron temperature are generated from BOLSIG+, a program aimed at solving for the transport properties of electrons.

The final required input, the electron number density N_e, is traditionally solved from the drift diffusion approximation of the conservation of momentum equation. Starting from the species continuity equation for electrons and neglecting diffusion, an equation for the time rate of change in electron density can be obtained:

$$\frac{dn(t)}{dt} = \alpha\left|n\mu_e\vec{E}\right| - \beta n^2, \tag{18}$$

where α determines the ionization frequency and β establishes the recombination rate of electrons. As equation 18 is function of the electric field experienced by a plasma, a complete system of equations will require equations 4, 6, 17, and 18 solved in a coupled manner. Equation 18 is also a function of empirical coefficients α and β which are both dependent on the pressure, p, of the system and can be given by

$$\alpha = Ap \exp\left(\frac{-Bp}{\vec{E}}\right), \tag{19}$$

and

$$\beta = C\left(\frac{300}{T_e}\right)^{\frac{1}{2}}, \tag{20}$$

where A, B, and C are tabulated values for air at atmospheric pressure. According to Raizer (*7*), appropriate values based on experimental measurements are: 38.1 (in·Torr)$^{-1}$ (15 [cm·Torr]$^{-1}$) for A, 927.1 V/(in·Torr) (365 [V/(cm·Torr)]) for B and 1.2×10^{-8} in^3/s (2×10^{-7}) cm^3/s for C.

3. Results and Discussion

In order to validate the accuracy of the model described in this paper, comparisons with data presented in Roupassov et al. (*1*) are provided. The experimental parameters that were mentioned and used in the circuit model are given in table 1.

Table 1. Experimental parameters.

h_a	0.016 in (0.4 mm)
h_d	0.01 in (0.3 mm)
ϵ_d	2.7
ϵ_a	1
A_n	0.05 in^2 (30 mm^2)
A_d	0.05 in^2 (30 mm^2)
V	50 kV
T	50 ns

An approximation of the 50-kV applied voltage pulse employed by Roupassov et al. (*1*) was considered for this study and is shown in figure 5.

For an E/N_a value of 4921 Td, BOLSIG+ calculates that the mean electron energy should be ~1.18×10^{-20} Btu (78 eV). Using this result, we calculated a peak number density of ~6.5×10^{17} 1/ft^3 (2.3×10^{19} 1/m^3) using equation 18 coupled with equation 4 where N_0, the number density before the pulse, was set equal to 2.83×10^{13} 1/ft^3 (10^{15} 1/m^3). The height of the plasma, h_a, was determined to be a constant 0.02 in (0.4 mm) over time based on experimental observations by Roupassov et al. (*1*). Thus using the calculated values for $N_e(t)$ and T_e as well as the experimentally observed h_a, we obtained a value of 2.3×10^{-6} Btu (2.6 mJ) by using equation 7. Gas heating results from Roupassov et al. (*1*) correspond to a total energy of 4.2 mJ that was imparted to air per 65 ns. Therefore, the approximate model presented in this study provides an answer with an absolute error of ~38%. The accuracy of higher-fidelity numerical models based on discretization of the drift diffusion equations has not been established.

4. Summary and Conclusions

A new lumped element circuit model was presented that is valid for any pulsed DC DBD plasma. In addition, an approximate expression was formulated to calculate the resistance value for the air gap as a function of the conductivity of the plasma. Results of the model were verified against a pulsed DC experiment conducted by Roupassov et al. (*1*), and an order of magnitude agreement was obtained for the energy imparted into the plasma in a homogeneous "hot spot" region. Future work will focus on extending the circuit model to account for the plasma region outside of the "hot spot" as well as determining the geometric dimensions of the plasma regions as functions of the applied voltage.

5. References

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